

THE GAUSS MAP OF A GENERIC HYPERSURFACE IN \mathbb{P}^4

CLINT McCrORY, THEODORE SHIFRIN & ROBERT VARLEY

Table of Contents

0. Introduction	689
1. The parabolic surface	694
2. The cusp hierarchy	698
3. Geometry of the cusp curve	704
4. The cubic threefold	716
5. Gauss-stable hypersurfaces	723
6. The transversality package	726
7. Classification of singularities	733
8. Genericity and deformations	736
9. Genericity and jet transversality	752

0. Introduction

The study of singularities of Gauss maps has played an important role in the development of algebraic geometry. Two beautiful examples of such work are the analysis of duals of cubic surfaces in projective 3-space \mathbb{P}^3 undertaken by Cayley and others in the nineteenth century, and Fano's investigation of the cubic threefold in the early part of this century. More recently, Andreotti exploited the Gauss map of the theta divisor to give a new solution to the Torelli problem for curves; Clemens-Griffiths, using a similar approach, proved that the cubic threefold is irrational.

Differential geometry and singularity theory are well adapted to the study of Gauss maps. Cartan's moving frames provide a dynamic tool which complements the static arguments of classical projective geometry. Arnold's theory of Lagrange and Legendre singularities has roots not only in classical mechanics, but also in projective geometry, for the Gauss map is the prototype of a Legendre map. Singularity theory provides a rich supply of analogies among singularities in different geometric contexts,

Received October 19, 1987. The first author was supported in part by National Science Foundation grant DMS-8603085, and the third author was supported in part by National Science Foundation grant DMS-8603281.