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MAGNETIC MONOPOLES ON THREE-MANIFOLDS

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Introduction

In this paper we will investigate magnetic monopoles on an oriented, complete Riemannian 3-manifold M. Basically the result is that we associate to M a collection of moduli spaces of solutions of the (magnetic) monopole equations, provided that M is not compact and that the Riemannian metric on M is 'good near infinity'. Topologically M may be the interior of any compact 3-manifold with boundary, but the metric on an end $\mathbf{R}_{>0} \times S$, S a boundary surface of M, should be approximately of the form $dl^2 + e^{2l}\, ds_S^2$.

This situation is much the same as that for an oriented, Riemannian 4-manifold, which has a collection of instanton moduli spaces associated to it. More specifically, we shall prove that monopoles exist under reasonable conditions, we compute the dimensions of the moduli spaces and study smoothness, orientability and asymptotic behavior. Having obtained these moduli spaces together with their basic properties, the next step would be to exploit the topology of the moduli spaces to define topological invariants for 3-manifolds, just as instanton moduli spaces give invariants for smooth 4-manifolds. This will be discussed in a forthcoming paper.

To carry nontrivial monopoles M should not be compact. This gives rise to hard analytical problems on the 3-manifold, such as those considered in the work of Taubes and Floer. To avoid this we shall exploit the fact that a monopole is a 'time'-invariant instanton on $M \times S^1$. Using the conformal invariance of the instanton equations, we find ourselves working on a conformal compactification X of $M \times S^1$, and studying S^1-invariant instantons on X. The fixed point set of the S^1-action on X now plays the role which the boundary of M played in a direct, 3-dimensional approach. In order to exploit as much as possible the available knowledge about instantons we carefully compare various definitions of S^1-invariance. This enables us to realize the monopole moduli spaces as submanifolds of instanton moduli spaces. This will be carried out in §1.

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