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DEFORMING THE METRIC ON COMPLETE RIEMANNIAN MANIFOLDS

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1. Introduction

In his paper [3] R. S. Hamilton introduced the evolution equation method which has proved to be very useful in the research of differential geometrical problems.

Using the evolution equation to deform the metric on any n-dimensional Riemannian manifold (M, g_{ij}):

$$\frac{\partial}{\partial t} g_{ij} = -2R_{ij}, \tag{1}$$

where R_{ij} is the Ricci curvature of M, the first important thing which we have to consider is the short-time existence of the solution of the evolution equation (1). In the case where M is a compact Riemannian manifold, Hamilton in [3] proved that for any given initial data metric g_{ij} on M the evolution equation (1) always has a unique solution for a short time. Therefore the short time existence problem of the evolution equation (1) was solved completely in the case when M is compact.

In the case where M is a noncompact complete Riemannian manifold, the short time existence problem of the evolution equation (1) is more difficult than the same problem for the compact case. Actually one cannot prove the short time existence of the evolution equation (1) for an arbitrary complete noncompact Riemannian manifold M; it is easy to find a complete noncompact Riemannian manifold (M, g_{ij}) on which the evolution equation (1) does not have any solution for an arbitrarily small time interval. Therefore to get the short time existence we have to make some assumptions on the curvature of M.

For a Riemannian manifold M with metric

$$ds^2 = g_{ij}(x)\, dx^i\, dx^j > 0,$$

we use $\{R_{ijkl}\}$ to denote the Riemannian curvature tensor of M and let

$$R_{ij} = g^{kl} R_{ikjl} \quad \text{and} \quad R = g^{ij} R_{ij} = g^{ij} g^{kl} R_{ikjl}$$

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