

HARMONIC MAPS INTO LIE GROUPS (CLASSICAL SOLUTIONS OF THE CHIRAL MODEL)

KAREN UHLENBECK

Dedicated to R. F. Williams

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This paper deals with two aspects of the algebraic structure of \mathcal{M} , the space of harmonic maps from a simply-connected 2-dimensional domain (either Riemannian or Lorentzian) into a real Lie group $G_{\mathbb{R}}$, the real form of a complex group G . In the language of theoretical physics, we study the classical solutions of the chiral model. In the first part of the paper (§§1-8) we construct a representation of the loop group $\mathcal{A}(S^1, G_{\mathbb{R}})$ on \mathcal{M} corresponding to the Kac-Moody Lie algebra of infinitesimal deformations observed by Dolan [8]. Here the main theorems are the description of the action on \mathcal{M} (Theorem 6.1) and the description of the action of a subgroup on the space of harmonic maps into Grassmannians (Theorem 8.3). In the second part of the paper (§§9-15) we restrict to a theory which applies only when Ω is a 2-dimensional, simply-connected, *Riemannian* domain and

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