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HARMONIC MAPS INTO LIE GROUPS (CLASSICAL SOLUTIONS OF THE CHIRAL MODEL)

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Dedicated to R. F. Williams

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This paper deals with two aspects of the algebraic structure of $\mathscr{M}$, the space of harmonic maps from a simply-connected 2-dimensional domain (either Riemannian or Lorentzian) into a real Lie group $G_{\mathbf{R}}$, the real form of a complex group G. In the language of theoretical physics, we study the classical solutions of the chiral model. In the first part of the paper (§§1–8) we construct a representation of the loop group $\mathscr{A}(S^1, G_{\mathbf{R}})$ on $\mathscr{M}$ corresponding to the Kac-Moody Lie algebra of infinitesimal deformations observed by Dolan [8]. Here the main theorems are the description of the action on $\mathscr{M}$ (Theorem 6.1) and the description of the action of a subgroup on the space of harmonic maps into Grassmannians (Theorem 8.3). In the second part of the paper (§§9–15) we restrict to a theory which applies only when Ω is a 2-dimensional, simply-connected, *Riemannian* domain and

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