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ANALYTIC EXTENSIONS OF THE ZETA FUNCTIONS FOR SURFACES OF VARIABLE NEGATIVE CURVATURE

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0. Introduction

The purpose of this note is to prove that for a compact C^∞ Riemannian surface of (variable) negative curvature the associated zeta function ς satisfies: $\varsigma(s)$ is nonzero and analytic on a half-plane $\mathrm{Re}(s) > h - \delta$ $(h, \delta > 0)$ except for a simple pole at $s = h$.

The result is well known in the special case that the surface has constant negative curvature (cf. [3], for example). For constant curvature surfaces one can use the Selberg trace formula, whose very existence seems to depend strongly on the Lie group construction of the surface. More generally it appears different techniques are required.

We adopt a dynamical viewpoint and study the associated geodesic flow. By an earlier result of the author (on more general Axiom A flows) we know that $\varsigma(s)$ can be extended *meromorphically* to a domain of the above form [9]. The difficulty is to show that no poles (other than at $s = h$) actually occur. For variable curvature geodesic flows we give a simple necessary condition for the occurrence of poles in this region (Lemma 4). The result follows by showing this condition is void (Lemma 5 and Theorem).

The proof that we give works at a slightly more general level than for geodesic flows. Our main result remains valid for the case of any transitive weak-mixing three-dimensional Anosov flow for which the stable and unstable horocycle foliations are continuously differentiable.

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1. Definitions and basic constructions

We begin by introducing basic material we shall need for the proof.

Let $\phi_t \colon M \to M$ be the geodesic flow on M, the unit tangent bundle of a compact C^∞ Riemannian surface S of strictly negative curvature. This

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