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GRIFFITHS' INFINITESIMAL INVARIANT AND THE ABEL-JACOBI MAP

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0. Introduction

In recent years, a body of theorems has accumulated about the locus of smooth surfaces of degree d in $\mathbf{P}^3$ possessing curves which are not complete intersections of the given surface with another surface. Let us call this the *Noether-Lefschetz locus.* The classical theorem of Noether and Lefschetz states that this locus has positive codimension when $d \geq 4$. A simple infinitesimal proof of this theorem is now known [2], every component is known to have codimension at least $d-3$ [5], and for $d \geq 5$, the only component having this codimension is the variety of surfaces containing a line ([4], [9]). There are still some fascinating open problems about this locus (see [8]).

For smooth hypersurfaces X in $\mathbf{P}^n$ in higher dimensions the situation is very different. If we look at codimension-one subvarieties on X, then the Lefschetz theorems show immediately that the Noether-Lefschetz locus is empty in all degrees. If we look at higher codimension subvarieties of X, then the question becomes quite interesting. C. Voisin has recently shown that a general 3-fold in $\mathbf{P}^4$ always possesses curves which cannot be obtained by intersecting X with a surface. There is, however, a beautiful conjecture of Griffiths and Harris [8] which is not contradicted by Voisin's example: *On a general 3-fold X of degree $d \geq 6$, the Abel-Jacobi map from algebraic 1-cycles on X homologically equivalent to zero to the intermediate Jacobian $J^2(X)$ is zero.*

This conjecture is still open. In this paper, we will lay out a three step program for proving it, and do the first two steps. This yields the following partial result:

Theorem 0.1. *For a general 3-fold of degree ≥ 6, the image of the Abel-Jacobi map on algebraic 1-cycles homologically equivalent to zero has image contained in the torsion points of the intermediate Jacobian.*

Here is a sketch of the argument: Let $\mathscr{X} \xrightarrow{\pi} B$ be a family of smooth $(2m+1)$-folds with $\mathscr{X}$ smooth and π everywhere of maximal rank. Let

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