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REGULARITY OF THE ALBANESE MAP FOR NONORIENTABLE SURFACES

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The function theory of a closed Riemann surface is fundamentally related to the Jacobi or Albanese map of the Riemann surface into a complex torus called the Jacobi variety of the surface. The Jacobi embedding theorem [2] states that this natural map is a smooth embedding and Abel's theorem [1] gives necessary and sufficient conditions, in terms of this mapping, for a divisor of the Riemann surface to be the divisor of a meromorphic function. Other important conformal information about the Riemann surface can be obtained from the embedding into its Jacobi variety. For example, Torelli's theorem [3] which describes the moduli space of Riemann surfaces in terms of the Jacobi map displays the deep interplay between the conformal structure of the Riemann surface and its Jacobi mapping.

The Albanese map, which in the case of Riemann surfaces is usually called the Jacobi map, is a holomorphic map of the Riemann surface into a flat complex torus, and this map induces an isomorphism between the first homology groups of the surface and the complex torus. In particular, the Albanese map is a harmonic map between the Riemann surface and a flat torus. For any closed Riemannian manifold M one can define a natural harmonic map $f\colon M \to A(M)$ where $A(M)$ is a flat torus $\mathbf{R}^n/\Lambda$ for some lattice Λ. The map f also has the property that it induces an isomorphism between first integral homology of M (modulo torsion) and the first integral homology group of $A(M)$. The map f is called the Albanese map of M and $A(M)$ is called the Albanese variety of M. A rigorous definition of $f\colon M \to A(M)$ is given at the beginning of the main body of the paper. For the moment it suffices to note that the Albanese map satisfies the following universal property: If $g\colon M \to T = \mathbf{R}^n/L$ is a harmonic map with $g(p_0) = 0$, and $f(p_0) = 0$, then $g = \pi \circ f$ for some "linear" homomorphism $\pi\colon A(M) \to T$.

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