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INTRINSIC CR NORMAL COORDINATES AND THE CR YAMABE PROBLEM

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1. Introduction

There is a deep analogy between the geometry of strictly pseudoconvex CR manifolds and that of conformal Riemannian manifolds. A CR manifold carries a natural hermitian metric on its holomorphic tangent bundle — the *Levi form* — which, like the metric on a conformal manifold, is determined only up to multiplication by a smooth function. The multiple is fixed by choosing a *contact form* — a real one-form annihilating the holomorphic tangent bundle. A CR manifold together with a choice of contact form is called a *pseudohermitian manifold.*

The simplest scalar invariant of a pseudohermitian manifold is the *pseudohermitian scalar curvature*, which we denote R, defined independently by S. Webster [15] and N. Tanaka [14]. The *CR Yamabe problem* is: *Given a compact, strictly pseudoconvex CR manifold, find a choice of contact form for which the pseudohermitian scalar curvature is constant.* In [6]–[8] we posed this problem and gave a sufficient condition for its solvability. The purpose of this paper is to show that "most" compact strictly pseudoconvex CR manifolds satisfy the sufficient condition, so that the CR Yamabe problem can almost always be solved. The precise statement of our result is Theorem A below.

Solutions to the CR Yamabe problem on a $2n+1$-dimensional CR manifold M are critical points of the functional

$$(1.1)\qquad \mathscr{Y}_M(\theta) = \frac{\int_M R\theta\wedge d\theta^n}{\left(\int_M \theta\wedge d\theta^n\right)^{2/p}}, \qquad p = 2+\frac{2}{n},$$

over the set of contact forms θ associated to the CR structure of M. In [7] we defined an invariant

$$\lambda(M) = \inf_\theta \mathscr{Y}_M(\theta),$$

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