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COMPONENTS OF MAXIMAL DIMENSION IN THE NOETHER-LEFSCHETZ LOCUS

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We will work over $\mathbf{C}$. Let

$$Y = \{\text{algebraic surfaces of degree } d \text{ in } \mathbf{P}^3\},$$
$$\Sigma_d = \{S \in Y \mid S \text{ smooth and } \operatorname{Pic}(S) \text{ is not generated by the hyperplane bundle}\}.$$

We will call Σ_d the *Noether-Lefschetz locus*. Some things that are known about Σ_d are:

(1) Σ_d has countably many irreducible components,

(2) For any irreducible component Σ of Σ_d,

$$d - 3 \leq \operatorname{Codim} \Sigma \leq \binom{d-1}{3}.$$

The upper bound on $\operatorname{codim} \Sigma_d$ is elementary, as this is just $h^{2,0}(S)$ (see [2]). The lower bound is more subtle and depends on fairly delicate algebraic considerations (see [4], [5]). One cannot do better for any $d \geq 3$, since the family Σ_d^0 of surfaces of degree d containing a line has codimension exactly $d-3$ in Y. For $d = 4$, the upper and lower bounds given in (2) coincide, so that every irreducible component of Σ_d has codimension one. For higher d, the following result was conjectured in [2]:

Theorem 1. *For $d \geq 5$, the only irreducible component of Σ_d having codimension $d-3$ is the family of surfaces of degree d containing a line.*

It should be noted that Theorem 1 was obtained independently by Claire Voisin [7].

Let Σ be an irreducible component of Σ_d having codimension $d-3$. As shown in [5], if $S = \{F = 0\}$ belongs to Σ, and $J_k(F)$ is the degree k piece of the Jacobi ideal of F, generated by the first partials F_0, F_1, F_2, F_3 of F, then:

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