NEW APPLICATIONS OF MAPPING DEGREES
TO MINIMAL SURFACE THEORY

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In [21], Tomi and Tromba showed how it was possible to use the degree theory of Smale [19] to solve the long open problem of proving that every smooth embedded curve in the boundary of a convex subset of $\mathbb{R}^3$ must bound an embedded minimal disk. Later Almgren and Simon [4] and Meeks and Yau [15] gave different proofs. In this paper we give other applications of degree theory to minimal surfaces. In particular, we show:

1. If $\Phi$ is an even constant coefficient parametric elliptic functional in $\mathbb{R}^3$ and $\gamma$ is a smooth embedded curve on the boundary of a strictly convex subset of $\mathbb{R}^3$, then $\gamma$ bounds an embedded $\Phi$-stationary and $\Phi$-stable disk. Furthermore, a generic such curve bounds an odd number of embedded $\Phi$-stationary disks and an even number of embedded $\Phi$-stationary surfaces of each other topological type.

2. Let $N$ be a smooth Riemannian 3-manifold with strictly mean convex boundary diffeomorphic to the 2-sphere. Suppose either that $N$ is not diffeomorphic to the 3-ball, or else that $N$ contains a compact minimal surface without boundary. Then there exists a sequence $D_i$ of embedded minimal disks in $N$ such that $\partial D_i \subset \partial N$, $\partial D_i$ converges to a smooth embedded curve $\gamma$, and the area of $D_i$ tends to infinity.

3. There exists a complete minimal hypersurface $M$ in $\mathbb{R}^n$ such that $M$ is singular, $M$ is not a cone, and $M$ is asymptotic at $\infty$ to an area minimizing cone $C$ that is regular except at the origin.

4. There exists a complete area minimizing hypersurface $M$ in $\mathbb{R}^n$ such that $M$ is asymptotic to an area minimizing cone $C$ that is regular except at the origin, but $M$ is not congruent to any leaf of the foliation of minimal hypersurfaces associated with $C$.

These results are proved in §§2, 3, 4, and 5, respectively. All depend on the preliminaries in §1, and §5 is a continuation of §4, but otherwise the sections are independent of each other. §6 discusses examples.

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