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ON THE SINGULARITIES OF THE SURFACE RECIPROCAL TO A GENERIC SURFACE IN PROJECTIVE SPACE

FELICE RONGA

1. Introduction

Let $S = S_f = \{[z_0, z_1, z_2, z_3] \in P^3 | f(z_0, z_1, z_2, z_3) = 0\}$ be a smooth surface in the complex projective space, where f is a homogeneous polynomial of degree n. Let P'^3 denote the space of hyperplanes in P^3, and $X_f = \{(a, h) \in S_f \times P'^3 | a \in h\}$, and define $p = p_f : X_f \to P'^3$ to be the natural projection. Denote by $\Sigma(p)$ the points of X_f where the derivative of p is not surjective. Among all the planes through $x \in S$ those tangent to S are special, so there should be no surprise that $\Sigma(p) = \{(a, h) | h = TS_a\}$, where TS_a denotes the tangent plane to S at a, and therefore that $p(\Sigma(p))$ is the surface reciprocal (or dual) to S.

Let A_n denote the vector space of homogeneous polynomials in three variables with complex coefficients, and P_n the projective space associated to A_n. Our purpose is to prove that for f in a nonempty Zariski open subset U_n of A_n the corresponding map p_f is excellent, which means that it has all the transversality properties required for these dimensions (Corollary 2.6). As a consequence, one has a complete description of all possible singularities of the surface reciprocal to S. Also, the fact that p is excellent provides global informations on the various singular loci, which have been exploited in [5] in order to justify some formulas of enumerative geometry found by G. Salmon [6] (the main proofs missed in [5] are provided here). Some work in the same direction was already done in [2], [3] and [4]. I am indebted to Clint McCrory for pointing out to me several mistakes in the first version of this paper.

We shall adopt the notation of [5]. In particular, given a smooth map $F: X \to Y$ and singularity types $\Sigma_1, \cdots, \Sigma_k$ applied to F, we set $M_k(\Sigma_1, \cdots, \Sigma_k) = \{x_1 \in X |$ there are $x_2, \cdots, x_k \in X$, $x_i \neq x_j$ for $i \neq j$ and $f(x_i) = f(x_j)\}$, and $N_k(\Sigma_1, \cdots, \Sigma_k) = f(M(\Sigma_1, \cdots, \Sigma_k))$. We shall denote by $J_0^k(\mathbb{C}^m, \mathbb{C}^n)$ the space of jets of order k of maps sending the origin to the origin.

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