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MORSE THEORY FOR LAGRANGIAN INTERSECTIONS

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Abstract

Let P be a compact symplectic manifold and let $L \subset P$ be a Lagrangian submanifold with $\pi_2(P, L) = 0$. For any exact diffeomorphism ϕ of P with the property that $\phi(L)$ intersects L transverally, we prove a Morse inequality relating the set $\phi(L) \cap L$ to the cohomology of L. As a consequence, we prove a special case of the Arnold conjecture: If $\pi_2(P) = 0$ and ϕ is an exact diffeomorphism all of whose fixed points are nondegenerate, then the number of fixed points is greater than or equal to the sum over the $\mathbf{Z}_2$-Betti numbers of P.

1. Introduction

Let (P, ω) be a symplectic manifold, i.e., P is a smooth manifold with a closed and nondegenerate 2-form ω. We can then assign to each smooth function

$$H : P \times \mathbf{R} \to \mathbf{R} : H(x,t) = H_t(x)$$

a family of vector fields X_t on P defined by

$$\omega(\cdot, X_t) = dH_t. \tag{1.1}$$

This is called the Hamiltonian vector field associated with the time dependent Hamiltonian H. If P is compact, then the differential equation

$$\frac{d}{dt}\phi_{H,t}(x) = H_t(\phi_{H,t}(x)) \tag{1.2}$$

with initial condition $\phi_{H,0}(x) = x$ defines a family of smooth diffeomorphisms of P, which also preserve the symplectic structure, i.e. for each $t \in \mathbf{R}$, we have $\phi_t^* \omega = \omega$.

In fact, the set

$$\mathscr{D} = \{\phi_{H,t} \mid t \in \mathbf{R} \text{ and } H \in C^\infty(P \times \mathbf{R})\} \tag{1.3}$$

turns out to be a subgroup of the group of symplectic diffeomorphisms of P.

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