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COMPACTIFICATION OF COMPLETE KÄHLER MANIFOLDS OF NEGATIVE RICCI CURVATURE

ALAN NADEL & HAJIME TSUJI

0. Introduction

The compactification of quotients of bounded symmetric domains with finite volume was obtained by Satake [19], Baily-Borel [5], Andreotti-Grauert [2] and Ash-Mumford-Rapoport-Tai [4]. Siu-Yau [21] obtained a compactification of complete Kähler manifolds with finite volume and sectional curvature pinched between two negative constants; their result may be regarded as a generalization of the compactification result for quotients with finite volume of bounded symmetric domains in the case of rank 1. In this paper, we investigate the problem of compactification of complete Kähler manifolds of negative Ricci curvature. Our main result is the following theorem.

Theorem 0.1. *Let M be a complete Kähler manifold of dimension m and let ω be the Kähler form. Assume the following.*

1. $\mathrm{Ric}(\omega) < 0$.
2. *M is very strongly $(m-2)$-pseudoconcave (cf. Definition 2.1).*
3. *The universal cover of M is Stein.*

Then M is biholomorphic to a quasiprojective manifold.

This theorem may be regarded as a generalization of the compactification results for quotients with finite volume of bounded symmetric domains of any rank (cf. §3).

Our proof depends on the weak Riemann-Roch theorem for L^2-plurigeneral (cf. §1) and the existence of Kähler-Einstein metrics (cf. §2). In §3, we shall give a purely differential geometric criterion for noncompact complete Kähler manifolds of finite volume to be quaisprojective.

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