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HOMOLOGY OPERATIONS ON INSTANTONS

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To Margaret and Susie

In this paper we study the topology of certain moduli spaces of instantons which are of central importance in the Yang-Mills gauge theory in mathematical physics. These moduli spaces, which arise naturally in the differential geometric formulation of the Yang-Mills theory, have been extensively studied in recent years by many people including Atiyah [2], [3], Atiyah, Hitchin and Singer [6], Atiyah, Drinfeld, Hitchin and Manin [5], Atiyah and Bott [4], Atiyah and Ward [8], Donaldson [17], [18], [19], [20], Drinfeld and Manin [21], [22], [23], Taubes [48], [49], [50], [51], [52], and Uhlenbeck [54], [55], using various techniques from algebraic geometry, complex manifold theory, global analysis and twistor theory. In particular Donaldson's seminal work [17], [20] has shown that the moduli spaces of instantons contain remarkable geometric information.

This rich influx of ideas into topology from other areas of mathematics has inspired advances using more classical topological techniques, as exemplified by the work of Fintushel and Stern [25], [26], and it is natural to ask if homotopy theoretic techniques may also be profitably applied to study moduli spaces of instantons. Of particular interest in this regard is the foundational paper of Atiyah and Jones [7], which gave the first homological information about these moduli spaces over the four-sphere for arbitrary instanton number k and formulated the basic topological questions in this subject. Specifically, Atiyah and Jones related these moduli spaces to components of well-known homotopy objects, namely configuration spaces and iterated loop spaces. In this paper we show that the disjoint union of these moduli spaces (where the union is taken over all positive instanton numbers k) behaves homologically like a four-fold iterated loop space (more precisely like a C_4 little cubes operad space in the sense of May [37]) with associated iterated loop space operations.

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