J. DIFFERENTIAL GEOMETRY
28 (1988) 383-405

GROMOV'S COMPACTNESS OF PSEUDO-HOLOMORPHIC CURVES AND SYMPLECTIC GEOMETRY

J. G. WOLFSON

In a beautiful recent paper [6] M. Gromov has introduced into symplectic geometry some new and ingenious techniques. In particular he associates to a symplectic structure a distinguished family of almost complex structures. Certain pseudo-holomorphic curves, that is, certain maps of a Riemann surface into the symplectic manifold which are "holomorphic" with respect to one of these almost complex structures, then provide global invariants of the symplectic structure. In this way, for example, Gromov proves that the open round ball $B_R \subset \mathbb{R}^{2n}$ of radius R cannot be symplectically diffeomorphic to an open subset of an ε-neighborhood of a nondegenerate $(2n-2)$ plane in $\mathbb{R}^{2n}$, unless $R \leq \varepsilon$ (see [6, Theorem 0.3.A]). The difficult analytic step in this work is the proof of the existence of suitable pseudo-holomorphic curves. As in any existence proof for a nonlinear elliptic partial differential system (or equation) the proof divides into two parts: (i) a proof of the openness of the space of solutions (often this part is accomplished using an "inverse function theorem") and (ii) a proof of the closeness or compactness of the space of solutions. Gromov accomplishes (i) using an index theorem computation and the Sard-Smale Implicit Function Theorem. The proof of (ii) involves some of the most delicate and beautiful parts of the paper. Although the space of pseudo-holomorphic curves is not, in any suitable topology, compact, by enlarging the space to allow for certain singularities Gromov proves the required compactness theorem. The proof of this result in [6] is rather brief and quite difficult. Fortunately, Pansu [9] has written notes clarifying and expanding the details of this proof. The proof's most interesting feature is that it is entirely geometric and at no time refers to results in differential equations. This has the unfortunate consequence of making the paper difficult for many mathematicians. Both to make Gromov's compactness result more accessible and to unify it with the many other compactness results in differential geometry, we give a new proof of this result using some ideas in partial differential equations due to Schoen and Uhlenbeck [11]. This work appeared

Received November 10, 1986 and, in revised form, July 10, 1987.