J. DIFFERENTIAL GEOMETRY
28 (1988) 309-339

THE HODGE COHOMOLOGY OF A CONFORMALLY COMPACT METRIC

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Abstract

The Hodge Laplacian acting on differential k-forms is examined for a class of complete Riemannian manifolds with negative sectional curvature near infinity. These manifolds have C^∞ compactifications on which the metric is conformal to one smooth up to the boundary with conformal factor ρ^{-2}, ρ a defining function for the boundary. An example is the Poincaré ball, which serves as a model throughout. The Schwartz kernel of a parametrix for the Laplacian is described for all degrees k except those near half the dimension of the manifold. Its asymptotics are determined in sufficient detail so that we may identify the L^2 harmonic spaces with the relative and absolute cohomology of the compactification for $k < (n-1)/2$ and $k > (n+1)/2$, respectively. In addition, we locate the essential spectrum of the Laplacian in each degree. The construction relies on a calculus of pseudodifferential operators well adapted to the type of degeneracy exhibited by the Laplacian at the boundary of the compactified manifold.

1. Introduction and geometric preliminaries

A. A complete simply-connected Riemannian manifold of negative curvature is diffeomorphic to Euclidean space. If in addition its metric is of bounded geometry then we might expect its Hodge Laplacian, which acts on differential forms, to behave much as the corresponding operator on the constant curvature model space $\mathbf{H}^n$. As we shall see, only sometimes is this the case.

The salient properties of the hyperbolic Laplacian are well known. Those which concern us here involve its spectrum on $L^2(dg_H)$, dg_H =hyperbolic measure, and the existence of bounded harmonic functions or forms with given "asymptotic" boundary values—including representation theorems in terms of these boundary values. Thus, if Δ_k denotes the Laplacian on k-forms, then Δ_k has no point spectrum unless $k = n/2$, in which case it has only an infinite dimensional eigenspace at zero (Dodziuk [5]). From Donnelly [6] its continuous spectrum is known to be unitarily equivalent to a finite sum of multiplication operators $a^2 + x^2$ acting on $L^2(\mathbf{R}^+, H)$, H an auxiliary Hilbert

Received January 28, 1987 and, in revised form, May 4, 1987.