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NEW MINIMAL SURFACES IN S^3

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Abstract

In this paper we construct new examples of compact imbedded minimal surfaces in S^3. We show some of these provide counterexamples to the conjecture that imbedded minimal surfaces separate S^3 into two domains of equal volume.

1. Introduction

We begin with the well-known tessellations of S^3 into cells having the symmetry of a Platonic solid in $\mathbf{R}^3$ and dihedral angle $2\beta_1$. Dividing a cell by its planes of symmetry we obtain as a fundamental region for the group of symmetries a tetrahedron with dihedral angles $\pi/2$, $\pi/2$, $\pi/2$, η, β_1, β_2 (see Table 1). The tetrahedron is determined by its dihedral angles.

TABLE 1

η, β_1, β_2	Cell Type	# of cells in tessellation	genus of constructed surfaces
$\pi/3, \pi/3, \pi/3$	Tetrahedral (Self-Dual)	5	6
$\pi/4, \pi/3, \pi/3$	Octahedral (Self-Dual)	24	73
$\pi/3, \pi/3, \pi/4$	Tetrahedral (or Cubical)	16(or 8)	17
$\pi/3, \pi/3, \pi/5$	Tetrahedral (or Dodecahedral)	600 (or 120)	601
$\pi/3, \pi/2, \pi/3$	Tetrahedral	2	3
$\pi/3, \pi/2, \pi/4$	Cubical	2	5
$\pi/3, \pi/2, \pi/5$	Dodecahedral	2	11
$\pi/4, \pi/2, \pi/3$	Octahedral	2	7
$\pi/5, \pi/2, \pi/3$	Icosahedral	2	19

To construct a minimal surface in S^3, we first find a minimal surface with boundary, called a "patch," within a tetrahedron (from Table 1) which intersects orthogonally all the plane-facrs of the tetrahedron in planar geodesics. From the patch we obtain a certain piece of the whole surface, called a "bone," by repeatedly reflecting "patches" through those plane-faces of the tetrahedron which are not contained in faces of the cell. Finally, we build the complete surface using reflections through faces of the cells (see Figures 1–3).[1]

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[1]All figures are stereographically projected to $\mathbf{R}^3$.