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THE LOW-DIMENSIONAL METRIC FOLIATIONS OF EUCLIDEAN SPHERES

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A *metric foliation* of a Riemannian manifold is a smooth partition into lower dimensional submanifolds which are locally everywhere equidistant, like for example, orbits of an isometric group action, or "parallel" hypersurfaces. In a general setting one has to allow singular focal leaves, but we will only consider the nonsingular case here. The leaves of a metric foliation are locally given as fibers of a Riemannian submersion which projects the induced metric of the normal bundle along the leaves isometrically to a metric on the quotient manifold. Although we are basically just dealing with Riemannian foliations for which the fixed metric is bundlelike [15], understanding the structure of metric foliations is primarily a problem in differential geometry that has a significant local component. Traditionally, in foliation theory only a transversal Riemannian structure is considered to be given on the ambient manifold.

Riemannian submersions satisfy highly overdetermined equations, and they do not exist for generic metrics, except with fibers of codimension 1. On the other hand, many important metrics, including in particular homogeneous metrics, admit an abundance of (usually nonhomogeneous) local Riemannian submersions with certain fiber dimensions, and sometimes, as in the case of constant curvature, with arbitrary fiber dimension. This makes the study of metric foliations in space forms especially attractive. Surprisingly enough, very little was known until recently.

In [6], we began a systematic investigation of k-dimensional metric foliations $\mathscr{F}^k$ in a constant curvature space Q_c^{n+k}, and we gave a complete analysis in the case $k = 1$. Here we continue this program and consider higher dimensional foliations. The general situation is quite complex, in particular locally. The global behavior of $\mathscr{F}$ depends critically on the sign of the curvature c. Our main result (Theorem 5.3) is concerned with spherical space forms. We classify the metric foliations $\mathscr{F}^k$ of the Euclidean sphere S^{n+k} for $k \leq 3$. They are all homogeneous, and moreover orbit foliations arising from a locally free

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