J. DIFFERENTIAL GEOMETRY
28 (1988) 37-57

ON THE HEAT OPERATORS OF NORMAL SINGULAR ALGEBRAIC SURFACES

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1. Introduction

Let X be a normal singular algebraic surface (over $\mathbf{C}$) embedded in the projective space $\mathbf{P}^N(\mathbf{C})$. The singularity set S of X is a finite set of isolated points. By restricting the Fubini-Study metric of $\mathbf{P}^N(\mathbf{C})$ to $\mathscr{X} = X - S$, we obtain an incomplete Riemannian manifold $(\mathscr{X}, g)$. Now consider the Laplacian $\Delta = \bar{\delta}\bar{d}$ acting on square-integrable functions on $(\mathscr{X}, g)$. Here $\bar{d}$ means the closure of the exterior derivative d acting on the smooth functions which are square-integrable, and whose images by d are square-integrable too. Also $\bar{\delta}$ means the closure of its formal adjoint δ acting on the smooth 1-forms which are square-integrable, and whose images by δ are square-integrable too. Then the purpose of this paper can be said to show the following.

Main Theorem. (1) *The Laplacian Δ is self-adjoint.*

(2) *The heat operator $e^{-\Delta t}$ is of trace class, and there exists a constant $K > 0$ such that*

$$\operatorname{Tr} e^{-\Delta t} \leq K t^{-2}, \qquad 0 < t \leq t_0. \tag{1.1}$$

Defining d_0 to be the exterior derivative d restricted to the subspace of smooth functions with compact supports, we have $\bar{\delta}^* = \bar{d}_0$ [4]. Hence (1) can be rewritten in the following way.

Assertion A. $\bar{d} = \bar{d}_0$.

In §5 we intend to prove this assertion, which is equivalent to (1). Thereby, we will prove (2) with $\Delta = \bar{\delta}\bar{d}_0$, the (self-adjoint) Laplacian of the (generalized) Dirichlet type (§§2–4).

In general, if a certain self-adjoint Laplacian on a certain Riemannian manifold has the *basic property* mentioned in (2), but replacing the 2 of t^{-2} by half of the real dimension of the manifold, then we say that the Laplacian has the *property* (BP). In using this expression, what we want to prove is stated as follows: $\Delta = \bar{\delta}\bar{d}_0$ has the property (BP). Let us transform this assertion (2)′ into a more convenient one.

Received May 5, 1986 and, in revised form, January 16, 1987. The author was supported by National Science Foundation grant MCS-8108814(A04).