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NONNEGATIVELY CURVED MANIFOLDS WITH SOULS OF CODIMENSION 2

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J. Cheeger and D. Gromoll have classified the complete noncompact manifolds of nonnegative curvature in dimensions ≤ 3 up to isometry (cf. [3]). This classification is partly based on the fact that for souls S of dimension 1 (respectively codimension 1), the manifold M is a locally isometrically trivial bundle over S (respectively a flat line bundle over S).

In dimension 4, an additional case may arise, namely $\dim S = \operatorname{codim} S = 2$. This situation is analyzed in §1, where it is shown that when S has codimension 2, there is a Riemannian submersion $\pi: M \to S$, or else the normal bundle $\nu(S)$ of S in M is flat with respect to the induced connection. Those M for which both conditions occur at the same time are the ones that split locally isometrically. Some results on total curvature follow. It turns out that the case where $\nu(S)$ is not flat is not as rigid as might be expected: in §2, the standard submersion metric on $S^3 \times_{S^1} \mathbf{R}^2$ is rather arbitrarily deformed while still retaining its nonnegative curvature. Finally, we show that given a metric of positive curvature on the n-sphere S, any 2-dimensional vector bundle over S admits a metric of nonnegative curvature with soul isometric to S.

1. Basic results

M will denote a complete noncompact manifold of nonnegative curvature with soul S. The reader is referred to [3] for the basic construction and main properties of souls, and to [6] for some facts about Riemannian submersions.

Lemma 1.1. *Let c: $[0, a] \to S$ be a piecewise smooth curve joining p and q in S, and suppose γ: $[0, \infty) \to M$ is a ray originating at p. If $u \in M_q$ denotes the parallel translate of $\dot\gamma(0) \in M_p$ along c, then $t \mapsto \exp_q(tu)$ is a ray originating at q.*

Proof. Since any piecewise smooth curve is a limit of broken geodesics, we may assume that c is a geodesic, and thus extendable to c: $\mathbf{R} \to S$. Carry

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