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PL MINIMAL SURFACES IN 3-MANIFOLDS

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Introduction

In [12], we studied the application of least weight normal surfaces to the equivariant decomposition theorems of 3-manifolds. The basic idea in [12] was to investigate ways in which such surfaces can intersect, and then to find new surfaces by cutting and pasting. Here we refine the notion of least weight normal surfaces to obtain piece-wise linear (PL) minimal surfaces in 3-manifolds. The main technique for analyzing PL minimal surfaces is by examining PL area under small variations of the surfaces. This yields new proofs of recent applications of least area (minimal) surfaces to the topology of 3-manifolds in [23] and [14], without going through the difficult existence theory for classical analytic minimal surfaces (cf., e.g., [22]).

Also we are able to show these PL minimal surfaces have most of the basic properties of analytic minimal surfaces. Some examples are as follows. We define *PL mean curvature* H for arbitrary normal surfaces and $H \equiv 0$ is necessary and sufficient for a normal surface to be PL minimal. In fact, there is a basic first variation argument for PL minimal surfaces, as in the analytic case. PL minimal surfaces satisfy a maximum principle and one can perform the crucial Meeks-Yau exchange and roundoff trick (see [16]–[20]). The notion of a convex boundary for a triangulated 3-manifold can be defined and so barrier arguments are possible (see, e.g., [18]). If two PL minimal surfaces intersect at a point of "tangency," then either the local picture is like a generalized saddle, as for analytic minimal surfaces (see [1]), or the surfaces locally coincide where they meet the 2-skeleton.

Finally the important results of Freedman-Hass-Scott [5] on the fundamental topological properties of least area surfaces go through in the PL case. Let

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