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COMPARING RIEMANNIAN FOLIATIONS WITH TRANSVERSALLY SYMMETRIC FOLIATIONS

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1. Introduction and result

In this paper we compare Riemannian foliations with transversally homogeneous foliations, where the model transverse structure is of the type of a compact symmetric space G/K. The datum needed for this comparison is a connection in the normal bundle, having similar properties as the canonical connection in the case of a transversally symmetric foliation. This similarity is most conveniently formulated in terms of the corresponding Cartan connections. For the symmetric model case the curvature of the Cartan connection vanishes. An almost transversally symmetric foliation is one where this curvature is small in an appropriate norm. In the spirit of Rauch's comparison theorem [15], and more specifically, the comparison theorem of Min-Oo and Ruh [13], we wish to conclude that this assumption already implies the existence of a transversally symmetric structure of type G/K. We succeed in doing so for tense Riemannian foliations with small mean curvature. Here the tenseness means that the mean curvature form of the Riemannian foliation is a basic 1-form. A weaker form of this result was announced in [6], where the mean curvature form was assumed to vanish.

The definitions required to formulate the precise result are in §2. The norms are defined in §5. In the following theorem we let $\mathfrak{g}$ and $\mathfrak{k}$ denote the Lie algebras of G and K respectively.

Theorem. *Let $\mathcal{F}$ be a transversally oriented Riemannian foliation of codimension $q \geqslant 2$ and basic mean curvature form κ on the compact oriented Riemannian manifold (M, g_M). Let G/K be an irreducible compact symmetric space of dimension q and semisimple $\mathfrak{g}$. There exists a constant $A > 0$ depending only on the Lie algebra $\mathfrak{g}$ and curvature bounds on M with the following property.*

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