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COMPACT HYPERSURFACES WITH CONSTANT SCALAR CURVATURE AND A CONGRUENCE THEOREM

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For an n-dimensional hypersurface M^n in the Euclidean space, we consider the rth mean curvature H_r, defined as the elementary symmetric polynomial of degree r in the principal curvatures of M^n. H_1, H_2, and H_n are the mean curvature, the scalar curvature, and the Gauss-Kronecker curvature respectively. The simplest global question concerning these geometric objects is the following:

> "*Given a compact hypersurface M^n embedded/immersed in the Euclidean space, such that H_r is constant for some $r = 1, \cdots, n$, is M^n a sphere?*"

The only solutions for this problem have been obtained in the cases $r = 1$ and $r = n$. If the mean curvature is constant and M^n is embedded, Aleksandrov [1] proved that M^n is a sphere. In the immersed case Hsiang, Teng, and Yu [3], and Wente [8], constructed nonspherical compact hypersurfaces in higher dimension and in $\mathbf{R}^3$, respectively. If the Gauss-Kronecker curvature is constant, then we conclude via the Hadamard theorem that M^n is strictly convex. But if M^n is strictly convex we know that $H_r =$ const., for some r, implies that M^n is a sphere (see Hsiung [4]). If $n = 2$ we obtain a classical result of Liebmann [6].

For the scalar curvature the problem has a special interest, which was proposed by Yau in [9].

In this paper we first prove that

> "*The sphere is the only compact hypersurface with constant scalar curvature embedded in the Euclidean space.*"

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