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THE UNIFORMIZATION THEOREM FOR COMPACT KÄHLER MANIFOLDS OF NONNEGATIVE HOLOMORPHIC BISECTIONAL CURVATURE

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Regarding the uniformization problem on compact Kähler manifolds of arbitrary dimensions, a very fundamental contribution was made by Mori [20] and Siu-Yau [24] when they affirmed the Frankel conjecture. They proved

Theorem (*special case of Mori* [20] *and Siu-Yau* [24]). *Let X be a compact Kähler manifold of positive holomorphic bisectional curvature. Then X is biholomorphic to the complex projective space $\mathbb{P}^n$.*

The theorem of Mori is stronger since he only assumed, in case the ground field is $\mathbb{C}$, that X is a compact complex manifold with ample tangent bundle and since his theorem applies equally to any algebraically closed field of arbitrary characteristic. Returning to the situation of compact Kähler manifold, it was natural to conjecture

(W_n) *Weak form of the uniformization conjecture for manifolds of nonnegative curvature*. Let X be an n-dimensional compact Kähler manifold of nonnegative holomorphic bisectional curvature and let $\tilde{X}$ be its universal covering space. Then $\tilde{X}$ is biholomorphic to $\mathbb{C}^k \times M$ for some nonnegative integer $k \leqslant n$ and some compact Hermitian symmetric manifold M.

In fact, (W_n) was in essence conjectured by Siu-Yau [24] prior to the solutions of the Frankel conjecture as part of a program to study the uniformization problem in higher dimensions. The purpose of the present article is to give a proof of the following stronger form of the uniformization conjecture for manifolds of nonnegative curvature, to be denoted by (S_n).

Main Theorem. *Let (X, h) be an n-dimensional compact Kähler manifold of nonnegative holomorphic bisectional curvature and let $(\tilde{X}, \tilde{h})$ be its universal covering space. Then there exist nonnegative integers k, $N_1, \cdots, N_l$ and irreducible compact Hermitian symmetric spaces $M_1, \cdots, M_k$ of rank $\geqslant 2$, such that*

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