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EUCLIDEAN DECOMPOSITIONS OF NONCOMPACT HYPERBOLIC MANIFOLDS

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In this paper, we introduce a method for dividing up a noncompact hyperbolic manifold of finite volume into canonical Euclidean pieces. The construction first arose in the setting of surfaces (see [7]), and in this case one gets a canonical cell decomposition of the surface and a canonical Euclidean structure. (The Euclidean structure, of course, is not complete.) The conformal structure underlying this Euclidean structure does not agree with the underlying hyperbolic structure, but the two conformal structures are probably not too distant (cf. Sullivan's theorem [5] for an analogous result).

This investigation arose from an attempt to understand the coordinates and cell decomposition of Teichmüller space due to Harer and Mumford [6] and independently to Thurston. Such coordinates and cell decompositions are also provided in [3] and [7]; in the latter, the action of the mapping class group on the coordinates is considered. We would like to thank J. Harer for the inspiration of his work and for several helpful remarks.

Our method is to work in Minkowski space and to represent a cusp by a point on the light-cone. The orbit of this point turns out to be discrete (even though the action of the group on the light-cone is ergodic), and we take the convex hull of the orbit. The boundary of this convex hull is decomposed into affine pieces, and one should think of the convex hull boundary as a kind of piecewise linear approximation to the upper sheet of the hyperboloid in Minkowski space. Each piece has a natural Euclidean structure. The suggestion that this might be possible first arose in a conversation between the authors and Lee Mosher. We thank Mosher for his contribution to this crucial idea. Comments by Brian Bowditch have also been helpful on a number of occasions. As a final credit, we wish to thank Bill Thurston. Much of this work as been discussed at various points with him, and the exposition has gained substantially from his comments.

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