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FOLIATIONS AND THE TOPOLOGY OF 3-MANIFOLDS. II

DAVID GABAI

Introduction

In this paper and its continuation [3] we investigate the following question: Let M be a compact oriented irreducible 3-manifold whose boundary is a torus. If N is obtained by filling ∂M along an essential curve α (i.e., N is obtained by attaching a 2-handle to ∂M along α and then capping off the resulting S^2 with a 3-cell), then does N possess a taut foliation? In this paper we consider the case when $H_2(M) \neq 0$ and in [3] we study the case when N is obtained by zero frame surgery on a knot k in S^3.

Using the existence of foliations on the filled manifolds we obtain a number of topological corollaries.

We now state (for reasons of clarity) a slightly less general version of the main result (Theorems 1.7, 1.8) of this paper.

Theorem. *Let M be an atoroidal Haken 3-manifold whose boundary is a torus and $H_2(M) \neq 0$. Let S be any Thurston norm minimizing surface representing a class of $H_2(M)$. Then with at most 1-exception (up to isotopy) the manifold N obtained by filling M along an essential simple closed curve in ∂M possesses a taut finite depth foliation $\mathscr{F}$ such that S is a leaf of $\mathscr{F}$ and the core of the filling is transverse to $\mathscr{F}$.*

Combining our main result with the work of Alexander, Reeb, Novikov, and Thurston (see [2, 2.5 and 2.8]) and some 3-dimensional topology we obtain the following results.

Corollary 2.14. *Let M be a connected sum of $M_1, \cdots, M_r$ where each M_i is either an oriented torus or sphere bundle over S^1. If k is a knot in M which does not lie in a 3-cell, then k is determined by its complement.*

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