J. DIFFERENTIAL GEOMETRY
26 (1987) 397–428

THE ORIENTATION OF YANG-MILLS MODULI SPACES AND 4-MANIFOLD TOPOLOGY

S. K. DONALDSON

1. Introduction

This paper has two separate purposes. The first is to modify the proofs of [3] and [6] (which considered simply connected manifolds) to obtain results on the intersection forms of 4-manifolds in the presence of fundamental groups. As an extension of the theorem of [3] we shall prove:

Theorem 1. *If X is a closed, oriented smooth 4-manifold whose intersection form*

$$Q : H^2(X; \mathbb{Z})/\text{Torsion} \to \mathbb{Z}$$

is negative definite, then the form is equivalent over the integers to the standard form $(-1) \oplus (-1) \oplus \cdots \oplus (-1)$.

In short, the result of [3] (Theorem A in [6]) extends without change to manifolds with arbitrary fundamental groups. For indefinite forms we shall prove:

Theorem 2. *Let X be a closed, oriented smooth 4-manifold with the following three properties*:

(i) $H_1(X; \mathbb{Z})$ *has no 2-torsion.*

(ii) *The intersection form Q on $H^2(X)/\text{Torsion}$ has a positive part of rank* 1 *or* 2.

(iii) *The intersection form is even.*

Then Q is equivalent over the integers to one of the forms

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

In short, Theorems B and C of [6] extend to manifolds with no 2-torsion in their first homology group.

Received March 26, 1986 and, in revised form, October 13, 1986.