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A STRICT MAXIMUM PRINCIPLE FOR AREA MINIMIZING HYPERSURFACES

LEON SIMON

It is a well-known consequence of the Hopf maximum principle that if M_1, M_2 are smooth connected minimal hypersurfaces which are properly embedded in an open subset U of an $(n+1)$-dimensional Riemannian manifold N, if $\overline{M}_1 \sim M_1$, $\overline{M}_2 \sim M_2 \subset \partial U$, and if M_1 lies locally on one side of M_2 in a neighborhood of each common point, then either $M_1 = M_2$ or $M_1 \cap M_2 = \varnothing$.

If we replace the hypothesis that $\overline{M}_j \sim M_j \subset \partial U$ by the hypothesis that the $(n-1)$-dimensional Hausdorff measure (i.e. $\mathscr{H}^{n-1}$) of $\overline{M}_j \sim M_j \cap U$ vanishes for $j = 1, 2$, then we still have either $\overline{M}_1 = \overline{M}_2$ or $M_1 \cap M_2 = \varnothing$. However this latter alternative leaves open the question of whether or not $\overline{M}_1 \cap \overline{M}_2 \cap U = \varnothing$, and it is this question which interests us here.

Here we settle the question affirmatively in the *area minimizing* case. Specifically (in Theorem 1 of §1) we show that $\overline{M}_1 \cap \overline{M}_2 \cap U = \varnothing$ if $M_1 \cap M_2 = \varnothing$ in case M_1, M_2 are the regular sets (in U) of integer multiplicity currents T_1, T_2 which are mass minimizing in U and which have zero boundaries in U. (Notice that in this case we have automatically that $\mathscr{H}^{n-1}(\overline{M}_j \sim M_j \cap U) = 0$, $j = 1, 2$, by the regularity theory for codimension 1 currents.)

Our interest in this problem originated from the paper [1], and the question was again raised in [2, Problem 3.4]. The proof of the result (given in §2) depends rather heavily on the main results of [1].

1. Preliminaries and statement of main result

The optimal version of the main theorem concerns codimension 1 integer multiplicity locally rectifiable currents T (called simply "locally rectifiable" in [3] and henceforth simply called "integer multiplicity" here) which are mass minimizing in an open set U of the smooth $(n+1)$-dimensional oriented

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