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SPLIT RANK AND SEMISIMPLE AUTOMORPHISM GROUPS OF G-STRUCTURES

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1. Introduction

This paper is a continuation of the investigation begun in [1], [3], [4] concerning the semisimple automorphism groups of G-structures on compact manifolds. In those papers we were concerned with semisimple groups that preserve a structure which is algebraic and which defines a volume density, i.e. where the structure group G is an algebraic subgroup of $\mathrm{SL}'(n,\mathbb{R})$, the matrices with $|\det| = 1$. (For higher order structures we assumed that G is an algebraic subgroup of $\mathrm{SL}'(n,\mathbb{R}) \cap \mathrm{GL}(n,\mathbb{R})^{(k)}$, the latter being the group of k-jets at 0 of diffeomorphisms of $\mathbb{R}^n$ fixing the origin.) One of the basic conclusions in the above papers is that for any simple noncompact Lie group H preserving such a G-structure, we must have that H locally embeds in G. (In fact a stronger assertion is proven. See the above papers and Theorem 2 below.) The main goal of the present paper is to consider the situation in which H is no longer assumed to define a volume density. In this case natural examples easily show that one cannot expect a local embedding of H in G. However, our main result asserts that a basic structural invariant of H must be visible in G. More precisely, we prove:

Theorem 1. *Let H be a semisimple Lie group with finite center and suppose that H acts smoothly on a compact manifold M so as to preserve a G-structure on M, where G is a real algebraic group. Then* $\mathbb{R}\text{-rank}(H) \leqslant \mathbb{R}\text{-rank}(G)$.

We recall that the $\mathbb{R}$-rank, or split rank, of a real algebraic group is the maximal dimension of an algebraic torus that is diagonalizable over $\mathbb{R}$. For a semisimple Lie group H, $\mathrm{Ad}(H)$ will be the connected component of the identity of a real algebraic group, and the $\mathbb{R}$-rank, or split rank, of H is defined to be the split rank of this real algebraic group. We shall also clear up

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