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IRRATIONALITY AND THE *h*-COBORDISM CONJECTURE

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I. Introduction

Underlying each smooth complex projective variety there is a compact differentiable manifold, and Hodge Theory links the topological cohomology of the space with its algebraic geometry. Despite this link there remains a striking difference between the successes of the two theories—the differential topology of manifolds and algebraic geometry of varieties—in problems of classification. For varieties there is a clearer picture in low dimensions, algebraic curves and surfaces, while for manifold topology high dimensions are more tractable. Smale's h-cobordism theorem gives a practical method for comparing the diffeomorphism types of simply-connected manifolds of dimension 5 or more, but for 4-manifolds the proof of the h-cobordism theorem breaks down.

In four dimenisons the first order Yang-Mills equations for gauge fields, like the Laplace equations of Hodge theory, give a path through partial differential equations between geometry and topology. On the one hand the solutions are, roughly speaking, generalizations of the holomorphic bundles over a complex surface. On the other hand their moduli spaces carry topological information. In this paper we exploit this path to define and calculate a new invariant for certain smooth 4-manifolds: we will see that the new invariant goes beyond the classical ones and our results indicate that there is a detailed structure in the differential topology of 4-manifolds quite analogous to that in the geometry of complex surfaces.

The manifold we used to test our invariant was discovered by I. Dolgachev in 1965 [4]. Dolgachev was motivated by the Castelnuovo-Enriques criterion

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