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# HOMOLOGY OF CLOSED GEODESICS IN A NEGATIVELY CURVED MANIFOLD

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## 0. Introduction

Let $M$ be a compact Riemannian manifold whose geodesic flow on the unit tangent bundle is of *Anosov type*. It is known that each free homotopy class of closed paths in $M$ contains a unique closed geodesic (W. Klingenberg [9]). In this paper, we show, by using a modified Perron-Frobenius theorem, that there exist infinitely many *prime* closed geodesics in each homology class in $H_1(M, \mathbb{Z})$. More precisely, we prove

**Theorem 1.** *If we denote by* $N(x, \alpha)$ *the number of prime geodesics* $\mathfrak{p}$ *in* $M$ *whose homology class is a given* $\alpha$ *and length* $l(\mathfrak{p}) \leqslant x$, *then the exponential growth rate* $\lim_{x\to\infty} x^{-1} \log N(x, \alpha)$ *is equal to the topological entropy* $h$ *of the geodesic flow* $(> 0)$.

The above theorem is intuitively anticipated from the fact that the fundamental group of $M$ has exponential growth, while $H_1(M, \mathbb{Z})$ has polynomial growth. In this view, one may ask whether there exist infinitely many closed orbits with a given homology class for a general Anosov flow $(X, \phi_t)$. It is known that the exponential growth rate of the number of closed orbits with respect to the period is always equal to the topological entropy. But one can easily construct an Anosov flow such that every homology class contains only finitely many closed orbits.

If $H_1(M, \mathbb{Z})$ is of finite order, Theorem 1 can be proven by means of dynamical $L$-functions, combining the idea of proof of the Chebotarev density theorem with the fact that there is a resemblance between prime closed geodesics and prime ideals in number fields (see [13], [17], [18]). In fact, we have

$$N(x, \alpha) \sim \left(\operatorname{Card} H_1(M, \mathbb{Z})\right)^{-1} \frac{e^{hx}}{hx}$$

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