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INSTABILITY OF THE LIOUVILLE PROPERTY FOR QUASI-ISOMETRIC RIEMANNIAN MANIFOLDS AND REVERSIBLE MARKOV CHAINS

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0. Introduction

It is shown by example that the Liouville property is not a quasi-isometry invariant of Riemann manifolds or for reversible Markov chains. Thus the example illustrates the subtleties involved in trying to understand the global function theory of Riemannian manifolds in terms of the behavior of discrete combinatorial models.

Let M be a manifold, ρ a complete Riemannian metric, and Δ_ρ the associated Laplacian operator. Many global function theoretic properties of Δ_ρ have geometric significance. This paper is concerned with the changes in the function theory which occurs as ρ is replaced by a quasi-isometrically equivalent metric τ; that is there exists a $C > 1$ such that for all $u \in TM_x$, for all x in M, we have $1/C < \rho(u,u)/\tau(u,u) < C$.

In parallel with manifolds we consider reversible Markov chains. These are defined by specifying a positive symmetric rate function $(a_{ij})_{i,j\in X}$ on a countable set X so that $\pi_i = \sum_{j\in X} a_{ij} < \infty$. These then admit the finite

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