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GAUGE THEORY ON ASYMPTOTICALLY PERIODIC 4-MANIFOLDS

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1. Introduction

S. K. Donaldson's theorem on the nonexistence of certain closed, smooth 4-manifolds [8] (and see [12]) has the surprising corollary that there exists an exotic smooth structure on $\mathbf{R}^4$. This corollary was deduced by M. Freedman using his machine [13] for analyzing topological 4-manifolds. The existence proof for this exotic structure is presented in [15], [12].

Subsequently, R. Gompf proved [15] that $\mathscr{R} = \{$oriented diffeomorphism classes of smooth manifolds which are homeomorphic to $\mathbf{R}^4\}$ has at least four elements. Freedman and L. Taylor [14] have produced a fifth element, and, recently, Gompf has shown that $\mathscr{R}$ contains a countable, doubly indexed family $\{\mathbf{R}_{m,n}\}_{m,n=0}^{\infty}$ of "exotic" $\mathbf{R}^4$'s [16], where, $\mathbf{R}_{0,0}$ is $\mathbf{R}^4$ with its standard smooth structure.

The primary purpose of this paper is to prove the following theorem.

Theorem 1.1. *There exists an uncountable family of diffeomorphism classes of oriented* 4*-manifolds which are homeomorphic to* $\mathbf{R}^4$.

The proof of the preceding theorem is a two part argument; the first part is basically topological in content, and the second part is basically analytical. The topological aspects of the proof were provided to the author by R. Gompf (see [16]).

Gompf relayed to the author (after an observation of R. Kirby) that Freedman's original existence proof realized an exotic $\mathbf{R}^4$, $\mathbf{R}$, as follows. In [13], Freedman constructs a closed, oriented topological 4-manifold, $|E_8 \oplus E_8|$, which is simply connected; and whose homology intersection form is the definite, nondiagonalizable (over $\mathbf{Z}$) unimodular symmetric form $E_8 \oplus E_8$. Donaldson [8] asserts that $|E_8 \oplus E_8|$ is not smoothable, but Freedman's surgery techniques show that $V \equiv |E_8 + E_8| \setminus \text{pt.}$ is smoothable. Now, according to Freedman there exists $\mathbf{R} \subset \mathscr{R}$, compact sets $K \subset V$ and $K_1 \subset \mathbf{R}$, and a

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