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PROJECTIVE STRUCTURES WITH FUCHSIAN HOLONOMY

WILLIAM M. GOLDMAN

A projective structure on a manifold S is a distinguished system of local coordinates modelled on a fixed projective space $\mathbf{P}$ in such a way that the local coordinate changes are locally projective. In this paper we will be mainly concerned with projective structures on manifolds of real dimension 2; thus $\mathbf{P}$ is either the complex projective line $\mathbf{CP}^1$ or the real projective plane $\mathbf{RP}^2$. If S is a topological surface then we shall speak of $\mathbf{CP}^1$-structures (resp. $\mathbf{RP}^2$-structures) on S; a manifold with a $\mathbf{CP}^1$-structure (resp. a $\mathbf{RP}^2$-structure) will be called a $\mathbf{CP}^1$-manifold (resp. an $\mathbf{RP}^2$-manifold).

It is well known that if M is a manifold with a projective structure modelled on a projective space $\mathbf{P}$, then there exists a pair (dev, φ) (unique up to projective automorphisms of $\mathbf{P}$), where dev: $\tilde{M} \to \mathbf{P}$ is a projective immersion, and φ is a homomorphism of $\pi = \pi_1(M)$ into the group of projective automorphisms of $\mathbf{P}$. The so-called developing map dev globalizes the coordinate charts defining the projective structure, while the holonomy homomorphism φ globalizes the coordinate changes. It is the purpose of this paper to classify projective structures on closed surfaces whose holonomy homomorphism is a fixed Fuchsian representation.

A *Fuchsian representation* of a discrete group π on $\mathbf{CP}^1$ is a faithful representation of π onto a discrete subgroup of $\mathrm{PSL}(2, \mathbf{C})$ preserving a disc Ω in $\mathbf{CP}^1$. Let φ be a Fuchsian representation φ of π on Ω. Using the Poincaré model for hyperbolic geometry, $\Omega/\varphi(\pi)$ has a natural hyperbolic structure, which we call the Fuchsian $\mathbf{CP}^1$-structure with holonomy φ. Conversely, every hyperbolic structure determines a Fuchsian $\mathbf{CP}^1$-structure in this way.

Similarly, suppose that φ is a representation of π in the group of projective transformations of $\mathbf{RP}^2$ (which we identify with $\mathrm{SL}(3, \mathbf{R})$). Then we say that φ is *Fuchsian* if φ is a faithful representation of π onto a discrete subgroup of

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