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THE YAMABE PROBLEM ON CR MANIFOLDS

DAVID JERISON & JOHN M. LEE

1. Introduction

The geometry of CR manifolds, the abstract models of real hypersurfaces in complex manifolds, has recently attracted much attention. This geometry is richest when the CR manifold is "strictly pseudoconvex," in which case there are many parallels with Riemannian geometry. (See the recent survey article by M. Beals, C. Fefferman, and R. Grossman [2] for a nice overview of these parallels.)

There are two complementary approaches to the study of CR geometry. The first is via the *Levi form*, a hermitian metric on complex tangent vectors; the second is via the *Fefferman metric*, a Lorentz metric on a natural circle bundle over the manifold.

Both of these geometric structures are determined only up to a conformal multiple by the CR structure. A choice of multiple of the Levi form is called a *pseudohermitian structure* on the manifold; such a choice also determines the multiple of the Fefferman metric.

The state of affairs suggests that, in order to find CR-invariant information, we proceed by analogy with conformal Riemannian geometry, in which a Riemannian metric is given only up to a conformal factor. A common strategy in conformal geometry is to choose a particular conformal representative for the metric which is normalized so as to simplify some aspect of the geometry. For example, the *Yamabe problem* on a conformal Riemannian manifold is to find a conformal representative for the metric that has constant scalar curvature. It is this problem that we generalize to CR manifolds in this paper.

An obvious analogue of the Yamabe problem for a CR manifold would be to find a pseudohermitian structure for which the associated Fefferman metric

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