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ONE-DIMENSIONAL GIBBS STATES AND AXIOM A DIFFEOMORPHISMS

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Abstract

We study the equilibrium statistical mechanics of one-dimensional classical lattice systems with exponentially decreasing interactions. For such systems, the Fournier transforms of pair correlation functions are meromorphic in a strip, and the residues of the poles can be expressed in terms of "Gibbs distributions." The latter are defined like Gibbs states, but without the positivity condition. Using symbolic dynamics, we can apply these results to Smale's Axiom A diffeomorphisms; the Gibbs distributions then become distributions in the sense of Schwartz on the manifold.

0. Introduction

The *Gibbs states* of equilibrium statistical mechanics are probability measures (on the space of configurations of an infinite system) which satisfy certain linear conditions.[1] We introduce here the more general concept of *Gibbs distributions*: they satisfy the same linear conditions as before, but are elements of a larger space than that of measures.[2] We determine a class of Gibbs distributions in the case of one-dimensional lattice systems with exponentially decreasing interactions. The interactions are allowed here to be complex. The main interest of this result is that it gives an expression for the residues of poles of the Fourier transform of the pair correlation function. While one-dimensional systems with short range interactions are in a sense trivial (they have no phase transition), they are useful in the analysis of certain differentiable dynamical systems on a manifold (Axiom A diffeomorphisms). We shall thus be able to study the correlation functions of these dynamical systems, and express the residues of the poles referred to, or *resonances*, in

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[1] See Dobrushin [5]–[7], and Lanford and Ruelle [11].

[2] For an early study of a special example see Gallavotti and Lebowitz [10].