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THE POLYSYMPLECTIC HAMILTONIAN FORMALISM IN FIELD THEORY AND CALCULUS OF VARIATIONS I: THE LOCAL CASE

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Abstract

An invariant geometric Hamiltonian formalism for multiple integral variational problems and field theories is presented. The formalism is based on the notion of a polysymplectic form, which is a vector valued generalization of symplectic forms. Hamiltonian equations, canonical transformations, Lagrange systems, symmetries, field theoretic momentum mappings and a classification of G-homogeneous field theoretic systems on a generalization of coadjoint orbits are investigated.

Introduction

The subject of this article is the Hamiltonian formalism for multiple integral variational problems and field theory.

In classical mechanics (see e.g. [1], [3]), the Hamiltonian formalism for single integral variational problems is the central structure and the base for the theory of symmetries, for statistical mechanics and for quantum mechanics. The geometric setting of the Hamiltonian formalism in terms of symplectic geometry lead to substantial progress, particularly in systems with symmetry groups, interaction models with gauge fields (minimal coupling), and the relation between classical and quantum systems. Thus Hamiltonian systems on symplectic manifolds are now the generally accepted fundamental frame for the dynamics of particle theories including quantum mechanics.

For field theory such a frame has been missing. Many of the quandries of quantum field theory may be due in part to the lack of a satisfactory

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