J. DIFFERENTIAL GEOMETRY
24 (1986) 343–372

ISOTOPY OF 4-MANIFOLDS

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The principal result of this paper is that the group of homeomorphisms mod isotopy (the "homeotopy" group) of a closed simply-connected 4-manifold is equal to the automorphism group of the quadratic form on H_2.

There is an analogy between simply-connected closed 4-manifolds and connected surfaces, in that they are both classified by simple algebraic-topological data. Surfaces are classified up to homeomorphism by the isomorphism class of the fundamental group. The 4-manifolds are classified up to homeomorphism by the isomorphism class of the intersection form and the Kirby-Siebenmann invariant in $\mathbf{Z}/2$ [3]. The analogy now extends to automorphisms, in that in both cases homeomorphisms are classified by the induced automorphism of the algebraic structure.

Other results include a "uniqueness" for handlebody structures on simply-connected 5-manifolds, the determination of $\pi_4(\mathrm{TOP}(4)/\mathrm{O}(4))$, and a pseudo-isotopy theorem for simply connected 4-manifolds with boundary.

1. Statements of results

Suppose M is a closed oriented (topological) manifold of dimension 4. Then intersections define a symmetric nonsingular bilinear form, denoted by λ, on H_2M. A homeomorphism of manifolds induces an isometry of H_2, and isotopic homeomorphisms induce the same isometry. Therefore there is a natural homomorphism from $\pi_0\,\mathrm{TOP}(M)$ (= homeomorphisms mod isotopy) to $\mathrm{Aut}(H_2M, \lambda)$.

1.1 Theorem. *Suppose M is a closed 1-connected 4-manifold. Then the natural homomorphism $\pi_0\,\mathrm{TOP}(M) \to \mathrm{Aut}(H_2M, \lambda)$ is an isomorphism.*

Freedman [3] has shown this to be onto. For injectivity we show that homeomorphisms which are equal on homology are isotopic, or equivalently that a homeomorphism inducing the identity on homology is isotopic to the

Received October 9, 1985 and, in revised form, June 2, 1986. The author was partially supported by the National Science Foundation.