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SINGULAR ANGULAR MOMENTUM MAPPINGS

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Abstract

We algebraically reduce the system consisting of a nonrelativistic particle moving in $\mathbf{R}^n$ with vanishing angular momentum $\mathscr{J}$. After analyzing the conical structure of the constraint set $\mathscr{J}^{-1}(0)$, we use algebraic geometric techniques to explicitly construct the reduced Poisson algebra of rotationally invariant observables. This procedure enables us to completely identify the effects of the singularity in $\mathscr{J}^{-1}(0)$ on the system. We then group-theoretically reduce the system and compare our results with those obtained algebraically.

0. Introduction

In celestial mechanics, rotational invariance allows one to eliminate four variables from Lagrange's equations. This procedure, Jacobi's celebrated "elimination of the node," has been generalized by Marsden and Weinstein [8] to the case when an arbitrary symmetry group acts on the phase space of a Hamiltonian system. The idea is as follows.

Consider a constraint of the form $\mathscr{J}=$ constant, where $\mathscr{J}$ is a momentum mapping for the group action. Then one may reduce the number of degrees of freedom of the system by dividing out the symmetries of the constraint set. Subject to certain technical assumptions, Marsden and Weinstein showed that the resulting "reduced phase space" of invariant states is in fact a symplectic manifold.

However, in many interesting situations the Marsden-Weinstein reduction procedure is not applicable and one must use instead the algebraic reduction technique of Śniatycki and Weinstein [12]. This yields a "reduced Poisson algebra" of invariant observables which contains all essential components of the reduced canonical formalism.

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