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LOWER BOUNDS FOR λ_1 ON A FINITE-VOLUME HYPERBOLIC MANIFOLD

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1. Introduction

Suppose M is an n-dimensional hyperbolic manifold having finite volume V, and denote by λ_1 the first positive element in the discrete spectrum for the problem $\Delta f + \lambda f = 0$ on M. If M is compact and $n = 2$, it is known [13] that there exists a constant $c > 0$, depending only on the genus of M, such that $\lambda_1 \geqslant cl$, where l is the total length of a smallest (in the sense of total length) collection of simple closed geodesics separating M. If M is compact and $n \geqslant 3$, it is known [12] that there exists a constant $c > 0$, depending only on n, and such that $\lambda_1 \geqslant cV^{-2}$. The 2-dimensional infinite volume case is discussed in [10].

In this paper we will discuss lower bounds for λ_1 in the finite volume case. Our results agree with those of [13] and [12] in cases covered by those papers. Our main purpose, however, is to illustrate a simple and general approach to this question, which depends on lower bounds for the first Dirichlet eigenvalues of some of the basic building blocks for hyperbolic manifolds.

In more detail, the Margulis lemma [2], [3], [6], [7], [9], [11], [14], implies that there exists $\varepsilon(n) > 0$, such that M is the union of two not necessarily disjoint subsets A and B, a thick and a thin part, for which:

1. A is not empty, and the injectivity radius at each point of A is greater than $\varepsilon(n) > 0$, so $V > v_1(n) > 0$. For $n \geqslant 3$, A is connected.

2. B is either empty or is a disjoint union of pieces, each of which is either:

(a) A closed embedded tubular neighborhood N of a simple closed geodesic γ, for which we may assume, by taking $\varepsilon(n)$ small enough, that the radius of N, i.e., the distance of any point on ∂N to γ, is greater than 1, or

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