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COMPACT SELF-DUAL MANIFOLDS WITH POSITIVE SCALAR CURVATURE

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Introduction

This article is the author's exercise in attempting to understand Hitchin's work on the classification of compact Kählerlian twistor space [21]. The motivation is from Langer's question on whether there is a self-dual metric on the connected sum of the complex projective planes.

A classification theorem in Riemannian geometry is the following [15], [22]: If X is a compact self-dual Einstein manifold with positive scalar curvature, then X is isometric to the Euclidean 4-sphere S^4 or the complex projective plane $\mathbf{CP}^2$ with the Fubini-Study metric.

We ask whether there is any self-dual metric with positive scalar curvature on a compact simply connected manifold which is *not* conformally equivalent to these standard metrics on S^4 or $\mathbf{CP}^2$. Our answer is contained in the following two theorems:

Theorem A. *Suppose that X is a compact simply connected self-dual manifold with positive scalar curvature. If the signature of X is equal to zero or one, then X is conformally equivalent to the Euclidean* 4*-sphere or the projective plane with Fubini-Study metric.*

Theorem B. (i) *There exists a one-parameter family of self-dual conformal classes on $\mathbf{CP}^2 \# \mathbf{CP}^2$, the connected sum of two complex projective planes with usual orientation.*

(ii) *Each of these conformal classes contains a metric with positive scalar curvature.*

(iii) *Each of these conformal classes has a two-dimensional torus as the identity component of the group of conformal transformation.*

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