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RIEMANNIAN MANIFOLDS ISOSPECTRAL ON FUNCTIONS BUT NOT ON 1-FORMS

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Introduction

For (M, g) a compact Riemannian manifold, let $\operatorname{spec}^p(M, g)$ denote the collection of eigenvalues, with multiplicities, of the associated Laplace-Beltrami operator acting on the space of smooth p-forms on M, $p = 0, 1, 2, \cdots, \dim(M)$. Two manifolds (M, g) and (M', g') will be said to be p-isospectral if $\operatorname{spec}^p(M, g) = \operatorname{spec}^p(M', g')$. Note that 0-isospectral manifolds are generally called "isospectral" in the literature. It is well known that $\operatorname{spec}^0(M, g)$ (i.e. the spectrum on functions) contains considerable information about the geometry of (M, g). Other information is known to be contained in the p-spectra for higher values of p. For example, Patodi [9] showed that $\operatorname{spec}^p(M, g)$, $p = 0, 1, 2$, together determine whether (M, g) has constant scalar curvature, whether it is Einstein, and whether it has constant sectional curvature. It would be of interest to determine whether for each k, the collection of all $\operatorname{spec}^p(M, g)$, $p = 0, \cdots, k$, contains more information than does $\operatorname{spec}^p(M, g)$, $p = 0, \cdots, k - 1$. The purpose of this article is to give an affirmative answer when $k = 1$, i.e., we give examples of manifolds which are 0-isospectral but not 1-isospectral.

The manifolds in our examples are Riemannian Heisenberg manifolds, i.e., compact quotients $\Gamma \setminus H^n$ of the $(2n + 1)$-dimensional Heisenberg group, with metrics g induced by left-invariant metrics on H_n. In [5], we gave sufficient conditions for two Riemannian Heisenberg manifolds to be 0-isospectral and constructed many examples. We will see that some of these examples are p-isospectral for all p while others are not 1-isospectral. We give evidence suggesting that these are the only two possibilities, i.e. that once the manifolds in these examples are 1-isospectral, they are also p-isospectral for all p. We

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