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THE HOMOLOGY OF THE MAPPING CLASS GROUP

EDWARD Y. MILLER

1. Introduction

The mapping class group Γ_g is the group of components of the groups $\mathrm{Diff}^+(S_g)$ or orientation preserving diffeomorphisms of a Riemann surface S_g of genus g. Since each component is contractible, there are natural isomorphisms of integral cohomology groups:

$$H^*\big(B\,\mathrm{Diff}^+(S_g); Z\big) = H^*\big(B\Gamma_g; Z\big). \tag{1.1}$$

In the context of complex analysis, Γ_g is called the Teichmuller group. It acts properly and discontinuously on the Teichmuller space T^{3g-3} with finite isotropy groups. The quotient of this action is the module space $\mathbf{M}_g$ of smooth algebraic curves of genus g. Consequently, there is an isomorphism of rational cohomology:

$$H^*\big(B\Gamma_g\colon A\big) = H^*\big(\mathbf{M}_g\colon Q\big). \tag{1.2}$$

In this paper we will show that $\mathbf{M}_g$, $B\Gamma_g$, and $B\,\mathrm{Diff}^+(S_g)$ get more and more complicated as the genus g tends to infinity. More precisely, we will prove:

Theorem 1.1. *Let $Q[z_2, z_4, z_6, \cdots]$ denote the polynomial algebra of generators z_{2n} in dimension $2n$, $n = 1, 2, 3, \cdots$. There are classes $y_2, y_4, \cdots, y_{2n}, \cdots$ with y_{2n} in the $2n$th cohomology group $H^{2n}(B\,\mathrm{Diff}^+(S_g); Z)$ such that the homomorphism of algebras sending z_{2n} to y_{2n}*

$$Q[z_2, z_4, \cdots] \to H^*(\mathbf{M}_g; Q) \cong H^*\big(B\,\mathrm{Diff}^+(S_g); Q\big)$$

*is an **injection** in dimensions less than $(g/3)$.*

These classes y_{2n} were first introduced by D. Mumford [7]. In the topological context, they are defined as follows:

Let $p\colon E \to B\,\mathrm{Diff}^+(S_g)$ be the universal S_g bundle with fiber S_g. Let d be the first Chern class of T_*, the tangent bundle along the fibers of the

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