J. DIFFERENTIAL GEOMETRY
23 (1986) 299–307

BRILL-NOETHER-PETRI WITHOUT DEGENERATIONS

ROBERT LAZARSFELD

Introduction

The purpose of this note is to show that curves generating the Picard group of a $K3$ surface X with $\mathrm{Pic}(X) = \mathbf{Z}$ behave generically from the point of view of Brill-Noether theory. In particular, one gets a quick new proof of Gieseker's theorem [5] concerning the varieties of special divisors on a general algebraic curve.

Let C be a smooth irreducible complex projective curve of genus g. One says that C satisfies *Petri's condition* if the map

$$\mu_0\colon H^0(A) \otimes H^0(\omega_C \otimes A^*) \to H^0(\omega_C)$$

defined by multiplication is injective for every line bundle A on C. Roughly speaking, this condition means that the varieties $W^r_d(C)$ of special divisors on C have the properties one would naively expect. Specifically, it implies that $W^r_d(C)$ is smooth away from $W^{r+1}_d(C)$, and that $W^r_d(C)$ (when nonempty) has the postulated dimension $\rho(r, d, g) =_{\text{def}} g - (r+1)\cdot(g - d + r)$. We refer to [1] for the definition of $W^r_d(C)$, and for a detailed discussion of Petri's condition and its role in Brill-Noether theory. One of the most basic results of this theory is Gieseker's theorem [5] that Petri's condition does in fact hold for the generic curve of genus g.

We prove here the following

Theorem. *Let X be a complex projective $K3$ surface, and let $C_0 \subset X$ be a smooth connected curve. Assume that every divisor in the linear system $|C_0|$ is reduced and irreducible. Then the general curve $C \in |C_0|$ satisfies Petri's condition.*

Received January 14, 1986. This work was partially supported by a Sloan Fellowship and National Science Foundation Grant DMS 84-05304.