J. DIFFERENTIAL GEOMETRY
23 (1986) 15-67

CURVATURE CHARACTERIZATION OF COMPACT HERMITIAN SYMMETRIC SPACES

NGAIMING MOK & JIA-QING ZHONG

In the study of complex manifolds the following conjecture is a well-known and natural analogue of the elliptic case of the uniformization theorem.

Conjecture I. Suppose X is a compact Kähler manifold of nonnegative holomorphic bisectional curvature and positive Ricci curvature. Then X is biholomorphic to a compact Hermitian symmetric space.

The special case, when X is of positive bisectional curvature and conjectured to be $\mathbf{P}^n$, is the Frankel conjecture, resolved simultaneously and independently by Mori [19] and Siu & Yau [22] in 1979 using very different methods. The general case of Conjecture I is at present still open. A related conjecture in case X is assumed to be Kähler-Einstein is the following.

Conjecture II. Suppose X is a compact Kähler-Einstein manifold of nonnegative holomorphic bisectional curvature and positive Ricci curvature. Then X is *isometric* to a compact Hermitian symmetric space.

The first efforts to resolve Conjecture II were due to Berger [3], who showed in 1966 that a compact Kähler-Einstein manifold of positive sectional curvature is isometric to $\mathbf{P}^n$ and equipped with the Fubini-Study metric (up to a scalar factor). This was reformulated by Goldberg and Kobayashi to the case of positive holomorphic bisectional curvature. Later, Gray [8] proved Conjecture II in 1973 under the stronger assumption of nonnegative Riemannian sectional curvature. He introduced on the unit sphere bundle of X a (degenerate) elliptic operator D and developed a Bochner-Kodaira formula for DR, R denoting the curvature tensor, to prove the vanishing of ∇R on X. The last property is the simplest characterization of locally symmetric spaces in terms of the curvature tensor. Apparently, there are serious difficulties in modifying Gray's argument to the general case of nonnegative holomorphic bisectional

Received December 14, 1984. This work was supported in part by National Science Foundation grant MCS-8108814 (A02).