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ON SOME AFFINE ISOPERIMETRIC INEQUALITIES

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In [15] it was shown that a certain intermediary inequality can be combined with the Blaschke-Santaló inequality to obtain a general version of the affine isoperimetric inequality (of affine differential geometry) and, in turn, that the equality conditions of this intermediary inequality can be used to obtain the Blaschke-Santaló inequality if one starts with this general version of the affine isoperimetric inequality. It was shown in [16] that another intermediary inequality can be combined with the Petty projection inequality to obtain a general version of the Busemann-Petty centroid inequality and, in turn, the equality conditions of this intermediary inequality can be used to obtain the Petty projection inequality if one starts with the general version of the Busemann-Petty centroid inequality. The two situations are remarkably similar. The similarity between the Blaschke-Santaló inequality and the Petty projection inequality is striking. However, no similar analogy appears to exist between the affine isoperimetric inequality and the Busemann-Petty centroid inequality. One of the objects of this article is to show that such an analogy does exist.

The setting for this article is Euclidean n-dimensional space, $\mathbf{R}^n$ ($n \geqslant 2$). We use $\mathscr{K}^n$ to denote the space of convex bodies (compact, convex sets with nonempty interiors) in $\mathbf{R}^n$, endowed with the topology induced by the Hausdorff metric. The support function of a convex body K will be denoted by h_K; i.e.,

$$h_K(x) = \operatorname{Max}\{x \cdot y \colon y \in K\},$$

where $x \cdot y$ is the usual inner product of x and y in $\mathbf{R}^n$. We will usually be concerned with the restriction of h_K to the unit sphere, S^{n-1}, in $\mathbf{R}^n$. The volume of a convex body K will be denoted by $V(K)$, and for the volume of the unit ball in $\mathbf{R}^n$ we use ω_n.

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