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A MODEL FOR CYCLIC HOMOLOGY AND ALGEBRAIC K-THEORY OF 1-CONNECTED TOPOLOGICAL SPACES

MICHELINE VIGUÉ-POIRRIER & DAN BURGHELEA

It was proven in [2] and [5] that the cyclic homology of a connected topological space with coefficients in a field of characteristic zero[1] k, $\widetilde{HC}_*(X; k)$, is isomorphic to $H_*(ES^1 \times_{S^1} X^{S^1}; k)$; it was also proven in [1] that the reduced algebraic K-theory of the 1-connected topological space X, tensored by k, $\tilde{K}_{*+1}(X) \otimes k$, is isomorphic to the reduced cyclic homology $HC_*(X; k)$. In this paper we describe a Sullivan minimal model of $ES^1 \times_{S^1} X^{S^1}$ in terms of a Sullivan minimal model of X (Theorem A). Our result completes the results of [10] which provides a Sullivan minimal model of X^{S^1} by giving a description of the Λ-minimal extension (in the sense of Halperin) of the fibration $X^{S^1} \to ES^1 \times_{S^1} X^{S^1} \to BS^1$, where X is 1-connected.

The model is effective enough to permit new calculations of the algebraic K-theory of X (tensored by k), where $X = X_1 \times X_2$ with X_2 a "rational" H-space or co-H-space and X_1 a product of complex or quaternionic projective spaces (or even more general) (see Theorem B and its corollaries). The model is also effective enough to contradict a conjecture of [4, p. 376]. T. Goodwillie has also calculated $K_*(CP^n) \otimes k$ by a different method.

The model is explicit enough to deal with the more subtle structures of cyclic homology (or algebraic K-theory) providing a fast (alternative) proof of a result of T. Goodwillie (Corollary 4).

This paper is organized as follows: In §1, we review the basic definitions and state the results; in §2, we give the proof of Theorem A; and, in §3, we give the proofs of the remaining results. We thank S. Halperin for useful conversations he had with the first named author about Theorem A. The present proof was influenced by this discussion.

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[1]Actually in any commutative ring.