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DEFORMING CONVEX HYPERSURFACES BY THE nTH ROOT OF THE GAUSSIAN CURVATURE

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Table of Contents

1. Introduction 117
2. Short time existence 118
3. Evolution of the metric and curvature 120
4. Preserving convexity 123
5. Convergence of the principal curvatures 125
6. Tso's theorem 136
7. Conclusion 137

1. Introduction

Recently, G. Huisken [4] studied the deformation of convex hypersurfaces in $\mathbf{R}^{n+1}$ by their mean curvature vector. In particular, he proved the following:

1.1 Theorem. *If the map F_0: $S^n \to M_0 \subset \mathbf{R}^{n+1}$ represents a strictly convex smooth hypersurface in $\mathbf{R}^{n+1}$, $n \geqslant 2$, then the initial value problem*

$$\text{(I)} \qquad \frac{\partial F}{\partial t}(x,t) = -H(x,t)\cdot\nu(x,t), \qquad F(x,0) = F_0(x), \quad x \in S^n,$$

has a unique solution on a maximum finite time interval $[0,T)$ such that the M_t's converge to a point as $t \to T$. Here H denotes the mean curvature and ν the outward normal of M. Moreover, if we let $\tilde{M}_t$ be M_t rescaled by a homothetic expansion so that $\mathrm{Vol}(\tilde{M}_t) = \mathrm{Vol}(M_0)$, then as $t \to T$ the $\tilde{M}_t$'s converge to a smooth hypersurface $\tilde{M}_T$ in the C^∞-topology. In fact $\tilde{M}_T$ is a round sphere.

The case $n = 1$ is a theorem of M. Gage and R. Hamilton [2]. Thus Theorem 1.1 may be considered as a generalization of their theorem to dimensions $n \geqslant 2$. Another possible generalization of their theorem to higher dimensions is the deformation of a convex hypersurface by its Gaussian curvature K.

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