J. DIFFERENTIAL GEOMETRY
22 (1985) 29–42

DECOMPOSITION THEOREMS FOR LORENTZIAN MANIFOLDS WITH NONPOSITIVE CURVATURE

JOHN K. BEEM, PAUL E. EHRLICH, STEEN MARKVORSEN
& GREGORY J. GALLOWAY

1. Introduction

The Toponogov Splitting Theorem [6] states that a complete Riemannian manifold (H, h) of nonnegative sectional curvatures which contains a line γ: $\mathbf{R} \to H$ (i.e., a complete absolutely minimizing geodesic) must be isometric to a product $\mathbf{R} \times H'$, the first factor being represented by γ. In [6] Cheeger and Gromoll gave a proof of this theorem stemming from their soul construction. Subsequently, Cheeger and Gromoll [5] were able to generalize this Riemannian splitting theorem to the case of nonnegative Ricci curvatures. In [17, p. 696], S. T. Yau raised the question of showing that a geodesically complete Lorentzian 4-manifold of nonnegative timelike Ricci curvature which contains a timelike line (i.e., a complete absolutely maximizing timelike geodesic) is isometrically the Cartesian product of that geodesic and a spacelike hypersurface.

Galloway [9] has recently considered this question for space-times which are spatially closed, i.e., which admit a smooth time function whose level sets are compact (smooth) Cauchy surfaces. Let (M, g) be such a globally hyperbolic space-time which satisfies the strong energy condition $\mathrm{Ric}(v, v) \geqslant 0$ for all timelike vectors v in TM. Suppose further that (M, g) contains a timelike curve which is both future and past complete and that for each $p \in M$, every null geodesic emanating from p contains a past and future null cut point to p.

Received August 25, 1984 and, in revised form, June 29, 1985. This material is based upon work of the first and second authors which was supported by the National Science Foundation under grant number DMS-840 1599 and was also supported in part by a grant from the Weldon Spring fund of the University of Missouri. The third author was supported in part by the Danish Natural Science Research Council.