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REAL KAEHLER SUBMANIFOLDS AND UNIQUENESS OF THE GAUSS MAP

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The main purpose of this paper is to answer the question: To what extent is a euclidean submanifold determined by its Gauss map? More precisely, let $f, \tilde{f}\colon M^n \to \mathbf{R}^{n+p}$ be two isometric immersions of a connected riemannian manifold whose Gauss maps into the Grassmannian G_{np} are congruent. When are f and $\tilde{f}$ congruent?

Classical examples of isometric noncongruent deformations with the same Gauss map are the associated families of minimal surfaces in $\mathbf{R}^3$. They are a special case of associated families which can be defined for certain real isometric immersions of Kaehler manifolds which we call *circular*. It will turn out that locally, all isometric immersions with the same Gauss map can be described in terms of circular submanifolds, whereas globally, additional phenomena arise.

In §1, we discuss circular submanifolds in spaces of constant curvature. This is related to work of Calabi, Lawson, and others on minimal surfaces. §2 deals with circular hypersurfaces. More generally, we classify all Kaehler submanifolds of real codimension 1, which is of independent interest. In the remaining two sections we show that all isometric immersions $M^n \to \mathbf{R}^{n+p}$ with congruent Gauss maps form a compact abelian group, and we compute its structure.

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1. The associated family of a circular immersion

Let M^n be a connected riemannian manifold of dimension n, and $f\colon M^n \to Q_c^{n+p}$ an isometric immersion into a complete simply connected space of constant curvature c. We will always assume that f is *substantial*, i.e. $f(M)$

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